**Semester One Examination, 2020**

**Question/Answer booklet**

**MATHEMATICS**

If required by your examination administrator, please

place your student identification label in this box

**SPECIALIST UNIT 3**

**Section One:**

# Calculator-free

WA student number: In figures

In words

Your name

|  |
| --- |
|  |

# Time allowed for this section Number of additional

Reading time before commencing work: five minutes answer booklets used

Working time: fifty minutes (if applicable):

# Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

# Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

# Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time  (minutes) | Marks available | Percentage  of  examination |
| Section One:  Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:  Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  |  |  |  | **Total** | 100 |

# Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

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WILL BE CUT OFF

1. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

1. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

1. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

1. It is recommended that you do not use pencil, except in diagrams.

1. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

1. The Formula sheet is not to be handed in with your Question/Answer booklet.

## Section One: Calculator-free 35% (52 Marks)

This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

## Question 1 (6 marks)

A system of equations, where 𝑏 is a real constant, is as follows:

𝑥 − 𝑦 + 3𝑧 = 11

𝑥 + 2𝑦 + 2𝑧 = 3

2𝑥 + 𝑏𝑦 + 4𝑧 = 8

1. Solve the system when 𝑏 = 3. (4 marks)

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1. Interpret the system of equations geometrically when 𝑏 = 4. (2 marks)

## Question 2

Polynomial 𝑃 is defined as 𝑃(𝑧) = 𝑧4 − 4𝑧3 + 14𝑧2 − 36𝑧 + 45.

1. Show that 𝑧 − 3𝑖 is a factor of 𝑃(𝑧). (2 marks)

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1. Solve 𝑃(𝑧) = 0, writing solutions in Cartesian form. (4 marks)

## Question 3

Functions 𝑓, 𝑔 and ℎ are defined as

4

𝑓(𝑥) = 𝑥 + 3, 𝑔 , ℎ(𝑥) = .

2 − 𝑥

1. Determine

* 1. ℎ ∘ 𝑔 ∘ 𝑓(6). (1 mark)

* 1. the defining rule for ℎ ∘ 𝑔 ∘ 𝑓(𝑥). (1 mark)

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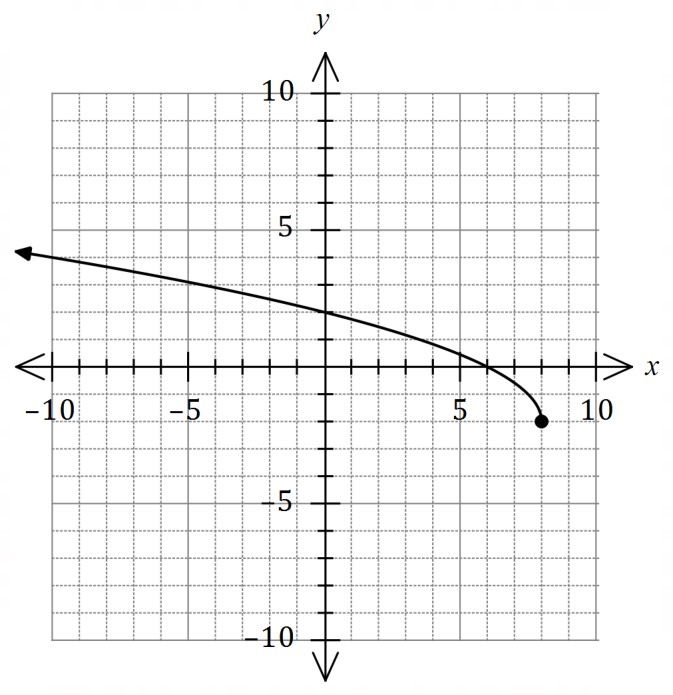
1. Determine the domain of ℎ ∘ 𝑔 ∘ 𝑓(𝑥). (2 marks)

1. Determine the range of ℎ ∘ 𝑔 ∘ 𝑓(𝑥). (2 marks) **Question 4**

The graph of 𝑦 = 𝑓(𝑥) is shown below.

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|  |  |
| --- | --- |
| (a) Draw the graph of 𝑦 = 𝑓−1(𝑥) on the same axes. | (3 marks) |

(b) Given that 𝑓, determine the defining rule for 𝑓−1(𝑥). (3 marks)

## Question 5 (8 marks)

−1 −3 3 0

Points 𝐴, 𝐵, 𝐶 and 𝐷 have position vectors ⃗𝑂𝐴⃗⃗⃗⃗ = ( 2 ), ⃗𝑂𝐵⃗⃗⃗⃗ = ( 1 ) , ⃗𝑂𝐶⃗⃗⃗⃗ = ( 4 ) and ⃗𝑂𝐷⃗⃗⃗⃗⃗ = (0).

4 7 −2 4

Note that |𝐮 × 𝐯| = |𝐮||𝐯| sin𝜃, where 𝜃 is the angle between 𝐮 and 𝐯 given by 𝜃 = cos−1(̂𝐮 ⋅ 𝐯̂).

1. Determine |⃗𝐴𝐵⃗⃗⃗⃗ × ⃗𝐴𝐶⃗⃗⃗⃗ | and use the result to explain why 𝐴, 𝐵 and 𝐶 are collinear. (5 marks)

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1. Determine the Cartesian equation of the plane containing all four points. (3 marks) **Question 6 (7 marks)**
2. Determine the complex cube roots of −1 in the form 𝑎 + 𝑏𝑖, where 𝑎, 𝑏 ∈ ℝ. (3 marks)

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1. Let 𝜔 be a complex cube root of unity, Im 𝜔 ≠ 0, so that 𝜔3 − 1 = 0.

* 1. Show that (𝜔 − 1)(𝜔2 + 𝜔 + 1) = 𝜔3 − 1 and hence explain why 𝜔2 + 𝜔 + 1 = 0.

(2 marks)

* 1. Simplify (2 + 5𝜔)(2 + 5𝜔2). (2 marks)

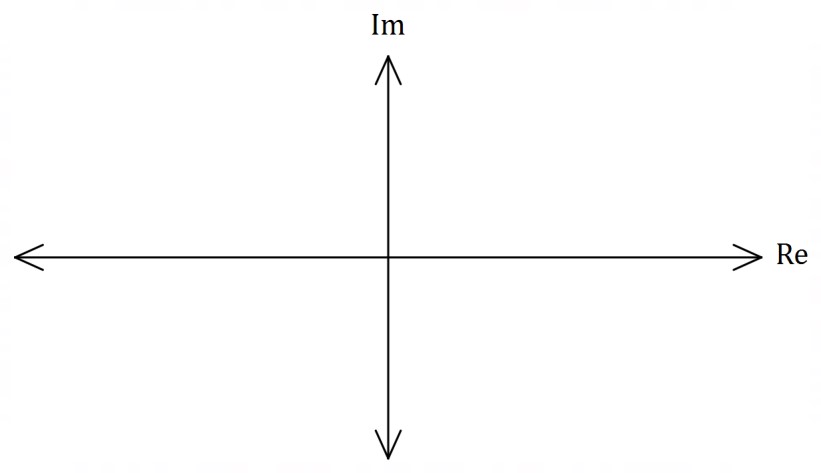
|  |  |
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| **Question 7**  The equation of line 𝐿 is  𝑥 − 2 𝑦 + 1 𝑧 − 1  = = .  2 −3 6 | **(6 marks)** |
| (a) Determine the vector equation of the line in the form 𝐫 = 𝐚 + 𝜆𝐛. | (2 marks) |

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(b) The diameter of sphere 𝑆 is the segment of line 𝐿 between 𝑥 = 2 and 𝑥 = 6. Determine

the equation of the sphere. (4 marks)

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| **Question 8**  The locus 𝐿1 of the complex number 𝑧 = 𝑥 + 𝑖𝑦 has equation |𝑧 − 6| = 2|𝑧 + 6|. | | **(7 marks)** |
|  |  |
| (a) | Show that 𝐿1 is a circle with equation 𝑥2 + 𝑦2 + 20𝑥 + 36 = 0. | (2 marks) |
| (b) | Sketch 𝐿1 on an Argand diagram. | (2 marks) |



|  |  |
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| Another locus 𝐿2 has equation 𝑤 ⋅ 𝑧̅ + 𝑤̅ ⋅ 𝑧 = 0, where 𝑤 = 4 + 3𝑖. |  |
| (c) Show that 𝐿2 is a tangent to 𝐿1. | (3 marks) |

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**End of questions**

Supplementary page

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Question number: \_\_\_\_\_\_\_\_\_

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